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Anomalous Low-Loss Transmission in a Gas-Confined Dielectric Waveguide for Millimeter and Submillimeter Wavelengths

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***Abstract*— A novel low-loss (gas-confined) dielectric waveguide for millimeter and submillimeter wavelengths was previously reported by the author. The waveguide consists of a thin dielectric tube separating an internal high-dielectric-constant gas from an external low-dielectric-constant gas. The attenuation constant of this form of waveguide usually increases with increasing tube thickness. The thick tube is indispensable for a mechanically stable waveguide. In this paper, anomalous low-loss transmission characteristics in a gas-confined dielectric waveguide with a thick tube are described. Some conditions are theoretically found where the attenuation constant of the waveguide with a thick tube is extremely low, due to tight field confinement within the internal gas. A qualitative explanation of the operation mechanism is also given.**

I. Introduction

Dielectric waveguides have received considerable interest in recent years for millimeter and submillimeter wavelengths transmission media. The advantage of the dielectric waveguides is that, in these wavelength regions, waveguide dimensions are of an order that can be easily fabricated and handled. However, the dielectric waveguide suffers from a fairly large attenuation. In order to bring the dielectric waveguide to practical use, it is most important to reduce transmission loss.

In designing a low-loss dielectric waveguide, two loss mechanisms, attenuation due to dielectric loss of materials and radiation loss from bends, should be simultaneously reduced. The transmission characteristics are closely dependent on waveguide field distribution. The attenuation of the dielectric circular rod waveguide is very small, if the wavefield energy exists almost entirely outside the dielectric rod [1]. A small curvature of the waveguide, however, will result in appreciable radiation losses. The attenuation is decreased at the expense of radiation loss.

To overcome these difficulties inherent in the dielectric waveguide, the author previously presented a novel approach to obtain a low-loss dielectric waveguide [2]. The waveguide consists of a thin dielectric tube which separates an internal high-dielectric-constant gas from an external low-dielectric-constant gas, as shown in Fig. 1. The characteristic feature of this waveguide is that the attenuation constant can be considerably reduced without incurring a radiation loss increase, because most of the power flows within the low-loss internal gas and not in the dielectric tube. The low-loss properties of the gas-confined dielectric waveguide have been demonstrated theoretically and experimentally.

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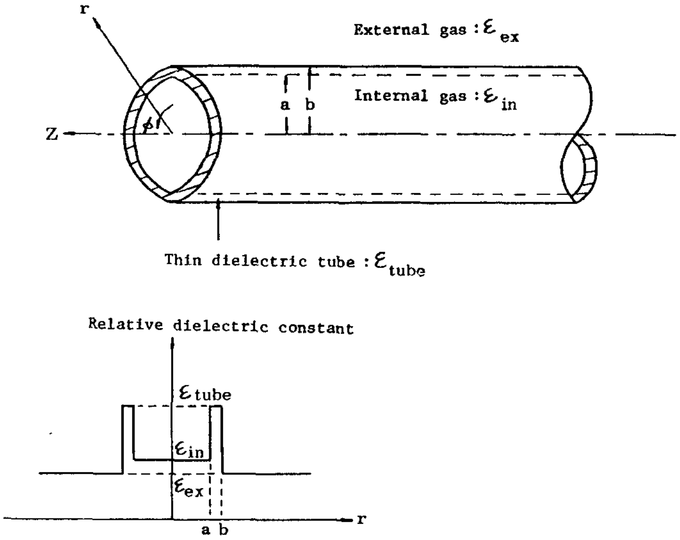

Fig. 1. Geometry and dielectric-constant profile of the gas-confined dielectric waveguide. The waveguide consists of a thin dielectric tube which separates an internal high-dielectric-constant gas from an external low-dielectric-constant gas.

Generally, gases, due to their low density, have much lower dielectric constants and much smaller absorption coefficients than solid dielectrics. If a dielectric tube is not so thin, most of the power is confined within the tube and not in the internal gas, and the waveguide suffers from a fairly large attenuation. The advantage of the gas-confined guide is brought about only when the tube is sufficiently thin. Thin dielectric tubes, however, involve problems with mechanical stability.

In this paper, the transmission characteristics of a gas-confined waveguide which consists of a relatively thick tube are presented. It is theoretically shown that there exist novel and anomalous conditions where most of the power travels within the internal gas and where propagation loss is extremely small. These phenomena were originally suggested by Marcatili [3]. His analysis was limited to extraordinary conditions. In this paper, the anomalous transmission characteristics are more accurately and clearly described.

II. Transmission Characteristics of Gas-Confined-Guide

The geometry and dielectric-constant profile of the gas-confined guide are shown in Fig. 1. Guided modes along the gas-confined guide can be analyzed in a standard manner. To clarify our results, discussions are limited to TE_{0n} modes propagation. Assuming a field with the time and z dependence of $e^{i(\omega t-\beta z)}$, axial field components of TE_{0n} modes can be expressed as follows:

$$H_z=\begin{cases} AI_0(\kappa r) \quad (\text{or } AJ_0(\kappa r)), & r\leq a \\ SJ_0(\sigma r)+TN_0(\sigma r), & a\leq r\leq b \\ CK_0(\gamma r), & b\leq r \end{cases} \tag{1}$$

where the factor $e^{i(\omega t-\beta z)}$ is omitted and

$$\begin{aligned}\kappa^2 &= \beta^2-\epsilon_{\rm in}k_0^2 \qquad (\text{or } \kappa^2=\epsilon_{\rm in}k_0^2-\beta^2)\\ \sigma^2 &= \epsilon_{\rm tube}k_0^2-\beta^2\\ \gamma^2 &= \beta^2-\epsilon_{\rm ex}k_0^2\end{aligned} \tag{2}$$

and k_0 is free-space wavenumber. Other field components ($E\phi$, H_r) are derived from H_z, using Maxwell's equations. The characteristic equation is derived by matching tangential-field components at $r=a$ and $r=b$. The existence of a nontrivial solution leads to an eigenvalue equation which determines the dispersion relation and field distribution of the waveguide.

The attenuation constant can be calculated by the perturbation method, assuming that the power lost per wavelength along the guide is small compared to the power traveling along the guide. Assuming that the propagation loss arises only from the dielectric tube and that gases are lossless, the attenuation constant is derived [2].

The attenuation constant is closely dependent on power flow distribution. The power in the internal gas, in the dielectric tube, and in the external gas are calculated and designated as $P_{\rm in}$, $P_{\rm tube}$, and $P_{\rm ex}$, respectively.

It is convenient to introduce the normalized frequency, which is most critical parameter in a dielectric waveguide. The average dielectric constant where $r<b$ is given by [4]

$$\bar{\epsilon}=\epsilon_{\rm in}\left(\frac{a}{b}\right)^2+\epsilon_{\rm tube}\left\{1-\left(\frac{a}{b}\right)^2\right\}. \tag{3}$$

The normalized frequency of a gas-confined waveguide is expressed as follows:

$$v=\frac{2\pi b}{\lambda_0}\sqrt{\bar{\epsilon}-\epsilon_{\rm ex}}\,. \tag{4}$$

When $\epsilon_{\rm tube}=\epsilon_{\rm ex}$, a gas-confined guide reduces to a conventional dielectric circular waveguide which consists of an internal high-dielectric-constant gas and an external low-dielectric-constant gas. The normalized frequency is defined as

$$v_{\rm rod}=\frac{2\pi a}{\lambda_0}\sqrt{\epsilon_{\rm in}-\epsilon_{\rm ex}}\,. \tag{5}$$

The cutoff wavelengths of TE_{01}, TE_{02}, and TE_{03} modes are calculated by equating $v=2.40$, 5.52, and 8.65, respectively, in a dielectric waveguide. When $\epsilon_{\rm ex}=\epsilon_{\rm in}$, a gas-confined guide reduces to a conventional O-guide [5]. The cutoff wavelengths (λ_c) are approximately expressed as follows:

$$\begin{aligned}\lambda_{c[01]} &= \sqrt{2}\,\pi a\sqrt{\frac{b}{a}-1}\,\sqrt{\epsilon_{\rm tube}-\epsilon_{\rm ex}}\,, \quad \text{for the } TE_{01} \text{ mode}\\ \lambda_{c[0n]} &= \frac{2(b-a)}{n-1}\sqrt{\epsilon_{\rm tube}-\epsilon_{\rm ex}}\,, \quad \text{for } TE_{0n} \text{ modes } n\geqslant 2.\end{aligned} \tag{6}$$

The radiation loss α_R is approximately expressed as follows [2]:

$$\alpha_R R=\frac{1}{2}(R\gamma)^{1/2}\exp\left(-\frac{R\lambda_0^2\gamma^3}{6\pi^2}\right) \tag{7}$$

where R is the radius of curvature and γ is given in (2).

III. Numerical Results

In order to demonstrate anomalous and interesting transmission characteristics of a gas-confined dielectric waveguide, numerical evaluation will be given.

The attenuation constant for the TE_{01} mode, calculated as functions of λ_0 and b, is shown in Fig. 2. The loss decreases with increasing λ_0 due to weak field confinement. The attentuation constant is reduced at the expense of radiation loss. The loss decreases with decreasing b, due to weak field confinement within the dielectric tube. If a tube is thin enough, most of the power is confined within the internal gas. The waveguide suffers from little radiation loss. The attenuation constant is closely dependent on the power flow distribution. The fractional powers in three parts (internal gas, tube, and external gas) are calculated as a function of λ_0, and shown in Fig. 3. The power in the tube increases with increasing b and with decreasing λ_0, which corresponds to the behavior of the attenuation constant in Fig. 2. These features of a gas-confined guide TE_{01} mode transmission are consistent with those of the HE_{11} mode transmission previously reported [2].

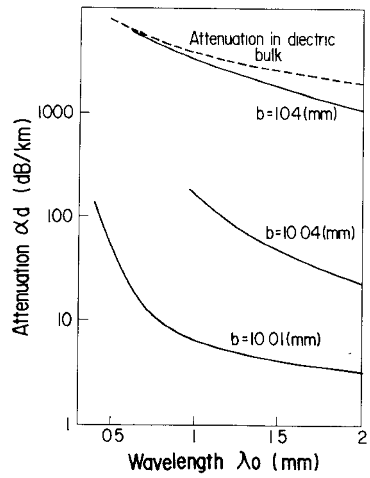

Fig. 2. Calculated attenuation constant in the gas-confined waveguide for TE_{01} mode as a function of λ_0 and b, where $a=10$ mm, $\epsilon_{\rm in}=1.01$, $\epsilon_{\rm tube}=2$, $\epsilon_{\rm ex}=1$, and $\tan\delta_{\rm tube}=10^{-4}$. The attenuation constant increases with increasing b and with decreasing λ_0.

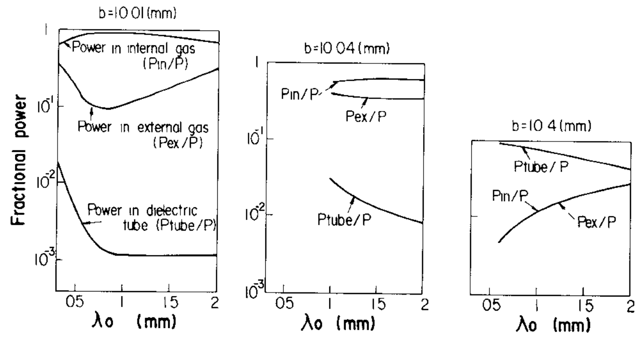

Fig. 3. Calculated fractional powers for TE_{01} mode in the internal gas, in the dielectric tube, and in the external gas as a function of λ_0 and b, where $a=10$ mm, $\epsilon_{\rm in}=1.01$, $\epsilon_{\rm tube}=2$, and $\epsilon_{\rm ex}=1$ The power in the tube increases monotonically with increasing b and with decreasing λ_0, which corresponds to the behavior of attenuation constant in Fig. 2. The power in the internal gas increases with decreasing b, which causes attenuation constant reduction. A very thin dielectric tube, however, involve problems with mechanical stability.

In order to bring this type of waveguide to practical use, a mechanically stable dielectric tube is necessary. The low-loss properties of the gas-confined guide obtained only when the tube is very thin and fragile.

To overcome the difficulty inherent in a gas-confined guide, a novel approach to obtain a low-loss transmission will be presented.

The attenuation constants for TE_{01}, TE_{02}, and TE_{03} modes as a function of λ_0 are shown in Fig. 4. The waveguide consists of a

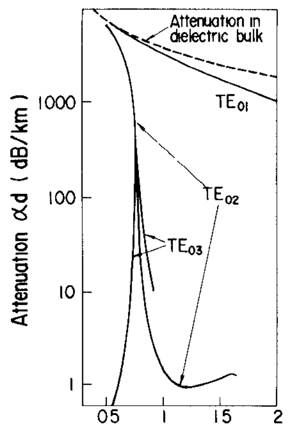

Fig. 4. Calculated attenuation constant in the gas-confined waveguide for TE_{01}, TE_{02}, and TE_{03} modes as a function of λ_0, where $a=10$ mm, $b=10.4$ mm, $\epsilon_{in}=1.01$, $\epsilon_{tube}=2$, $\epsilon_{ex}=1$, and $\tan\delta_{tube}=10^{-4}$. The waveguide consists of a relatively thick tube ($b-a=0.4$ mm). The attenuation constant for TE_{01} mode increases monotonically with decreasing λ_0. The attenuation constant for TE_{02} mode, however, decreases with decreasing λ_0 from 1.6 to 1.2 mm, and that for TE_{03} mode decreases with decreasing λ_0 from 0.78 to 0.5 mm.

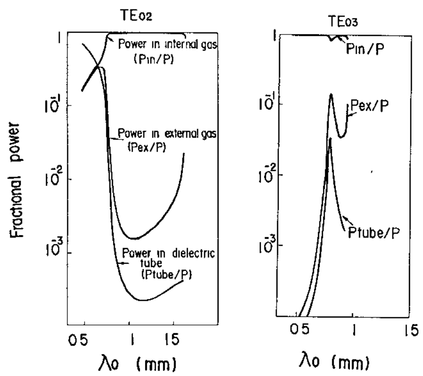

Fig. 5. Calculated fractional powers in the internal gas, in the dielectric tube, and in the external gas as a function of λ_0 for TE_{02} and TE_{03} modes, where $a=10$ mm, $b=10.4$ mm, $\epsilon_{in}=1.01$, $\epsilon_{tube}=2$, and $\epsilon_{ex}=1$. It should be noted that the power in the dielectric tube for TE_{02} mode decreases with decreasing λ_0 from 1.6 to 1.2 mm and that for TE_{03} mode decreases with decreasing λ_0 from 0.78 to 0.5 mm, which corresponds to the behavior of the attenuation constants in Fig. 4. The guided power is almost perfectly confined within the internal gas in the above conditions, which simultaneously reduces the two losses, attenuation due to dielectric loss of a tube and radiation loss from bends.

relativity thick tube (0.4 mm). The attenuation constant for the TE_{01} mode increases monotonically with decreasing λ_0. The attenuation constant for the TE_{02} mode, however, decreases with decreasing λ_0 from 1.6 mm to 1.2 mm, and that for the TE_{03} mode decreases with decreasing λ_0 from 0.78 mm to 0.5 mm. The fractional powers in three parts as a function of λ_0 are shown in Fig. 5. [7]. The power in the dielectric tube for the TE_{02} mode decreases with decreasing λ_0 from 1.6 mm to 1.2 mm, and that for the TE_{03} mode decreases with decreasing λ_0 from 0.78 mm to 0.5 mm, which corresponds to the behavior of the attenuation constants in Fig. 4. It is especially noted that the guided power is almost perfectly confined within the internal gas in the above conditions, which simultaneously reduces dielectric loss and radiative loss.

Calculated radiation loss as a function of λ_0 is shown in Fig. 6. The radiation loss rapidly decreases with decreasing λ_0. It should be noted that the radiation losses for the TE_{02} and TE_{03} modes

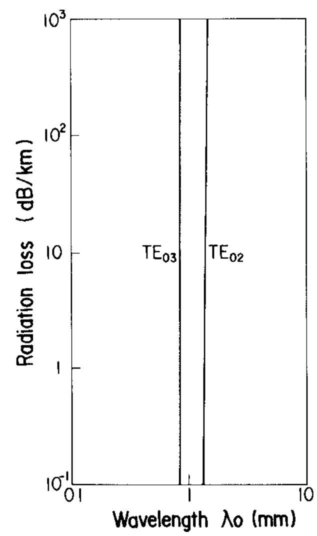

Fig. 6. Calculated radiation loss as a function of λ_0 for TE_{01}, TE_{02}, and TE_{03} modes, where $a=10$ mm, $b=10.4$ mm, $\epsilon_{in}=1.01$, $\epsilon_{tube}=2$, $\epsilon_{ex}=1$ and bending radius $R=20$ m. The radiation loss rapidly decreases with decreasing λ_0. It should be noted that the radiation losses for TE_{02} and TE_{03} modes are low enough in the wavelengths regions where the attenuation constants are very low, due to tight field confinement within the internal gas. The radiation loss for TE_{01} mode is below 0.1 dB/km.

are low enough in the wavelengths regions where the attenuation constants are very low, due to tight field confinement within the internal gas. The radiation loss for the TE_{01} is below 0.1 dB/km in this region.

Some conditions are found where the attenuation constant of the waveguide with a relatively thick tube is extremely low due to tight field confinement within the internal gas.

IV. Discussion

The novel low-loss transmission characteristics, which are quite suitable to a practical waveguide line, will qualitatively be discussed. As the dielectric constants of the internal gas and the tube are higher than that of the external gas, as shown in Fig. 1, the delivered power is confined within the internal gas and the tube. If the tube is thick, the power is almost confined within the tube and transmission properties are similar to O-guide propagation. If the tube is thin, the power is considerably confined within the internal gas and transmission properties are similar to those of a dielectric rod waveguide consisting of an internal high-dielectric-constant gas and an external low-dielectric-constant gas. Anomalous transmission characteristics are closely related with the difference between the above two waveguide properties.

The electric field distributions for the TE_{01} and TE_{02} modes are calculated as a function of distance from the origin (r), as shown in Fig. 7. The electric field in the tube ($a \leqslant r \leqslant b$) monotonically increases with decreasing λ_0 for the TE_{01} mode. For TE_{02} mode, however, the electric field distribution is complicated and strongly dependent on λ_0. When λ_0 is small ($\lambda_0=0.5$ mm), the strongest electric field is in the tube. When λ_0 approaches to 1 mm, the field distribution is drastically changed. The electric field is mostly confined in the internal gas, and the field in the tube becomes fairly weaker. These features are also found for the TE_{03} mode.

The electric field $E\phi(r \leqslant a)$ changes as $I_1(\kappa r)$ when λ_0 is small and as $J_1(\kappa r)$ when λ_0 is near the cutoff wavelength. κ is calculated as a function of λ_0 for the TE_{01}, TE_{02}, and TE_{03} modes as shown in Fig. 8. Solid lines correspond to the case when

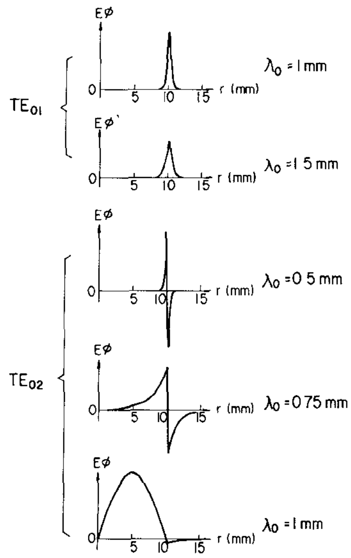

Fig. 7. Calculated electric field distribution as a function of λ_0 for TE_{01} and TE_{02} modes, where $a=10$ mm, $b=10.4$ mm, $\epsilon_{in}=1.01$, $\epsilon_{tube}=2$, and $\epsilon_{ex}=1$. The electric field in the tube monotonically increases with decreasing λ_0 for TE_{01} mode. For TE_{02} mode, however, the electric field distribution changes drastically. When λ_0 is small ($\lambda_0=0.5$ mm), the field is almost confined in the internal gas, and the field in the tube becomes weaker. These features are consistent with the attenuation constant in Fig. 4. The anomalous transmission characteristics are explained by the difference of cutoff frequencies between O-guide and dielectric rod guide.

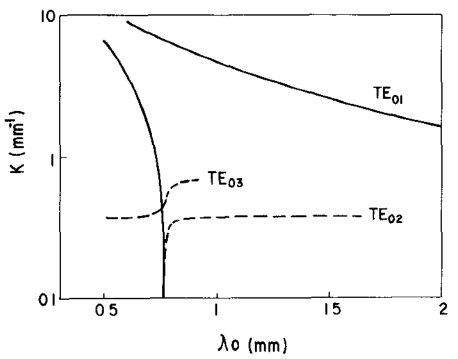

Fig. 8. Calculated wavenumber κ. Solid lines correspond to the case that $H_z=AI_0(\kappa r)$ in (1) and $\kappa^2=\beta^2-\epsilon_{in}k_0^2$ in (2). Dotted lines correspond to the case that $H_z=AJ_0(\kappa r)$ in (1) and $\kappa^2=\epsilon_{in}k_0^2-\beta^2$ in (2). For TE_{02} and TE_{03} modes, there exist wavelengths regions where $\kappa \sim 0.38$ mm^{-1} on the dotted lines and $E_\phi \sim 0$ at $r=a=10$ mm. For TE_{01} mode, however, the region where $\kappa \sim 0.38$ mm^{-1} is not found.

the propagation constant $\beta \geq \sqrt{\epsilon_{in}}\,k_0$ and the field changes as $I_1(\kappa r)$. Dotted lines correspond to the case when $\sqrt{\epsilon_{in}}\,k_0 \geq \beta \geq \sqrt{\epsilon_{ex}}\,k_0$ and the field changes as $J_1(\kappa r)$. For the TE_{02} and TE_{03} modes, there are some wavelengths regions where $\kappa \sim 0.38$ mm^{-1} on the dotted lines. E_ϕ changes as $J_1(\kappa r)$, then E_ϕ approaches to zero when $r=a=10$ mm ($\kappa r=3.8$), which is consistent with the behavior of E_ϕ when $\lambda_0=1$ mm for the TE_{02} mode in Fig. 7. It is found that $\kappa a \sim 3.8$ when 0.8 mm$\leq \lambda_0 \leq 1.6$ mm for the TE_{02} mode, and 0.5 mm$\leq \lambda_0 \leq 0.7$ mm for the TE_{03} mode. These wavelengths regions approximately correspond to low-loss transmission in Fig. 4 and tight field confinement within the internal gas in Fig. 5.

Cutoff wavelengths of the TE_{01}, TE_{02}, and TE_{03} modes are shown in Table I. Cutoff wavelengths of a gas-confined guide are approximately determined by v. If the dielectric constant difference between ϵ_{in} and ϵ_{ex} is neglected, the cutoff wavelengths are determined by (6). If $\epsilon_{tube}=\epsilon_{ex}$, they are calculated by v_{rod}.

For the TE_{01} mode, the cutoff wavelength of O-guide is fairly

TABLE I
CUTOFF WAVELENGTHS OF TE_{0n} MODES

		O-guide	Dielectric rod waveguide	Gas-confined waveguide	
Dielectric constant profile		$\varepsilon_r=\varepsilon_{ex}$, $r<a$; $=\varepsilon_{tube}$, $a<r<b$; $=\varepsilon_{ex}$, $b<r$	$\varepsilon_r=\varepsilon_{in}$, $r<a$; ε_{ex} $a<r$	$\varepsilon_r=\varepsilon_{in}$, $r<a$; ε_{tube}, $a<r<b$; ε_{ex}, $b<r$	
Mode	TE_{01}	8.9 mm	2.61 mm	7.91 mm	9.3 mm
	TE_{02}	0.8	1.14	3.34	1.64
	TE	0.4	0.73	2.20	0.92
Approximation		Eq (6)	Eq (5)	Eq (4)	Rigorous eigen value equation
			Vrod (or V)=2.40, TE_{01}; 5.52, TE_{02}; 8.65, TE_{03}		

$a=10$ mm, $b=10.4$, $\varepsilon_{in}=1.01$, $\varepsilon_{tube}=2$, $\varepsilon_{ex}=1$

The cutoff wavelength of O-guide for TE_{01} mode is fairly larger than those for TE_{02} and TE_{03} modes. The normalized frequency defined by the dielectric constant difference between the internal and external gases, v_{rod}, at the TE_{01} mode cutoff wavelength of O-guide is much smaller than v_{rod} at the TE_{02} or TE_{03} mode cutoff wavelengths of O-guide: $v_{rod}=0.71$, TE_{01}; $v_{rod}=7.9$, TE_{02}; $v_{rod}=15.7$, TE_{03}. The cutoff wavelength of the gas-confined guide for TE_{01} mode is similar to that of O-guide. The cutoff wavelengths of the gas-confined guide for TE_{02} and TE_{03} modes are fairly larger than those of O-guide. These facts suggest that gas-confined waveguide transmission for TE_{02} or TE_{03} modes is quite different from O-guide transmission.

larger than that of circular rod waveguide and is approximately coincident with that of the gas-confined guide. For the TE_{02} and TE_{03} modes, the cutoff wavelengths of O-guide are smaller than those of the gas-confined guide. This fact suggests that O-guide propagation is no more observed in the region between the cutoff wavelengths of the O-guide and the gas-confined guide. When λ_0 is larger than the cutoff wavelength of O-guide, the field is scarcely confined within the tube, and it is strongly confined within the internal gas. The reason is that the normalized frequency v_{rod}, which is defined by the dielectric constant difference between the internal and external gases, is fairly large. For example, $v_{rod}=7.9$ when $\lambda_0=0.8$ mm (= the TE_{02} mode cutoff wavelength of O-guide). These wavelengths regions may extend from the cutoff wavelength of O-guide to that of dielectric rod waveguide. They are 0.8–1.14 mm for the TE_{02} mode, and 0.4–0.73 mm for the TE_{03} mode. These regions approximately correspond to small attenuation constant in Fig. 4 and tight field confinement within the internal gas in Fig. 5.

V. CONCLUSION

Novel low-loss propagation supported in a gas-confined dielectric waveguide is described. The waveguide consists of a thin dielectric tube separating an internal high-dielectric-constant gas from an external low-dielectric-constant gas. In order to bring the waveguide to practical use, the following requirements should be satisfied: 1) attenuation due to dielectric loss of materials is small, 2) radiation loss from bends is small, and 3) the waveguide consists of a relatively thick tube so that it can be mechanically stable.

Transmission characteristics of TE_{0n} modes are theoretically investigated. The attenuation constant, power flow distribution and radiation loss are calculated. Some operation conditions are found where these requirements are simultaneously satisfied. Even if the tube is relatively thick, the power is almost perfectly confined within the internal gas in these conditions, which drastically reduces attenuation and radiation loss. These anomalous transmission characteristics are explained by the difference of cutoff frequencies between O-guide and circular rod waveguide. A gas-confined dielectric waveguide will have practical application to low-loss transmission lines and high-Q resonators in the submillimeter wavelengths region.

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Composite Dielectric Waveguides with Two Elliptic-Cylinder Boundaries

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Abstract—**It is shown that the propagation constants of composite dielectric waveguides with two different elliptic-cylinder boundaries, such as the recent single-polarization optical fibers, are computable by the point-matching method. Numerical results are shown for various combinations of the dielectric constants.**

I. INTRODUCTION

Composite dielectric waveguides, or dielectric-rod structures composed of a few dielectric materials, are expected to be applicable to the transmission of optical waves and microwaves.

Composite dielectric waveguides having two semicircular dielectric regions were discussed in a previous paper [1] where the propagation constants of various transmission modes were computed by applying the point-matching method to interface conditions between two dielectric regions and a microwave model experiment to confirm numerical results was described.

Recent investigations of single-polarization optical fibers [2], [3], have attracted our attention to multicylindrical boundary structures such as an elliptical core with circular cladding and a circular core with elliptical cladding. The detailed analysis of wave propagation characteristics along such dielectric waveguides is complicated compared with one-boundary structures [4]–[6], but it is important.

Ferdinandoff and Bulgarien discussed dielectric waveguides with elliptical cross sections and derived approximate characteristic equations for some extreme cases [7]. However, they have not indicated any numerical data of waveguide characteristics or computational feasibility to support his method.

This paper shows the applicability of the point-matching method to the modal analysis of some composite dielectric waveguides with two elliptic-cylinder boundaries including two crossing boundaries.

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II. THE METHOD OF ANALYSIS

The cross-sectional view of various dielectric waveguides is shown in Fig. 1(a)–(j). The first structure, a circular dielectric rod, as shown in Fig. 1(a), is a historical one which was analyzed by Hondros and Debye seventy years ago [4] with the method of the separation of variables. Electromagnetic fields inside and outside of the rod in this case were expressed by using Bessel's and modified Bessel's functions, respectively.

The second structure in Fig. 1(b), a rectangular dielectric rod, can no longer be treated with the method of the separation of variables. Goell [8] applied the point-matching method to the analysis in order to satisfy interface conditions imposed on electromagnetic fields, and used a linear combination of Bessel's functions to express the fields.

This method was employed to analyze the wave propagation along optical fibers with deformed boundaries such as a chipped circle as shown in Fig. 1(c) [5]. The numerical results of this analysis were also confirmed by a microwave model experiment [5].

A composite dielectric waveguide as shown in Fig. 1(d), was analyzed by modifying the point-matching method so as to treat three dielectric regions [1]. A microwave model experiment indicated data supporting the consequent analytical results [1].

Optical fibers of a circular core with an elliptical cladding and those of an elliptical core with a circular cladding as shown in Fig. 1(e) and (f), were fabricated by Kaminow *et al.* [2] and Matsumura *et al.* [3] for maintaining a state of polarization over an extended length. The cross sections of core-cladding structures as shown in Fig. 1(e)–(h) belong to a class of composite dielectric waveguides. These structures can be expressed by a combination of two different elliptic-cylinder boundaries as shown in Figs. 2 and 3.

First, we employ the circular cylindrical coordinate systems (r, θ, z) and assume the propagation factor $\exp(j\omega t - j\beta z)$ in each field function. Then, the fundamental wave equations are given by

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}-\beta^2+k_i^2\right)\begin{Bmatrix}E_{zi}\\ H_{zi}\end{Bmatrix}=0, \qquad i=1,2,3 \tag{1}$$

where the suffix 1 denotes the central dielectric region with the highest dielectric constant, the suffix 3 the outside dielectric region, and the suffix 2 the remaining dielectric region. Therefore, the dielectric constants of the three regions are related by

$$\epsilon_1 > \epsilon_2 \qquad \epsilon_1 > \epsilon_3. \tag{2}$$

Then, the wavenumber of each region is given by

$$k_i^2 = \omega^2 \epsilon_i \mu_0, \qquad i=1,2,3. \tag{3}$$

The propagation constant β should be in the range

$$k_1 > \beta > k_3. \tag{4}$$